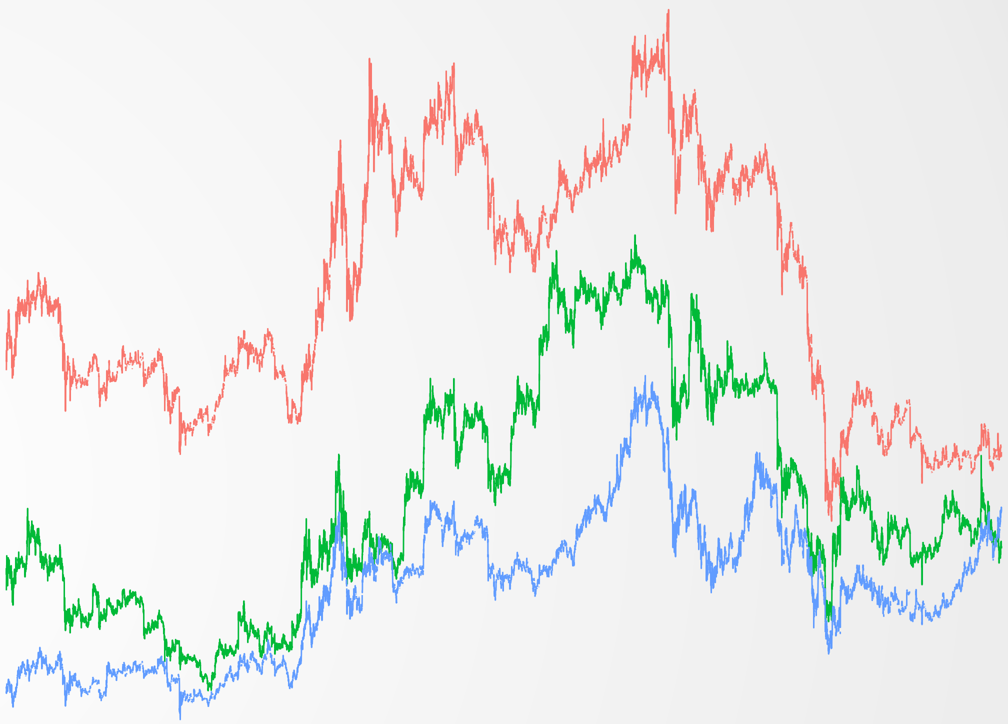
# Contagion dynamics in high frequency - modeling shock impacts in cryptocurrency markets



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# Intro to crypto - Bitcoin bigger than largest DAX stock

**market cap**

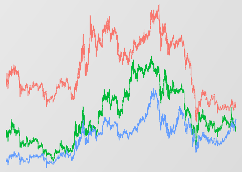
**€ 200 bn. *** — **€ 160 bn. ***

## Many cryptocurrencies

- BTC - Bitcoin
- LTC - Litecoin
- DSH - Dash
- XMR - Monero
- …

*** September 30, 2020**

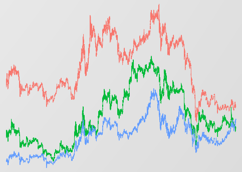




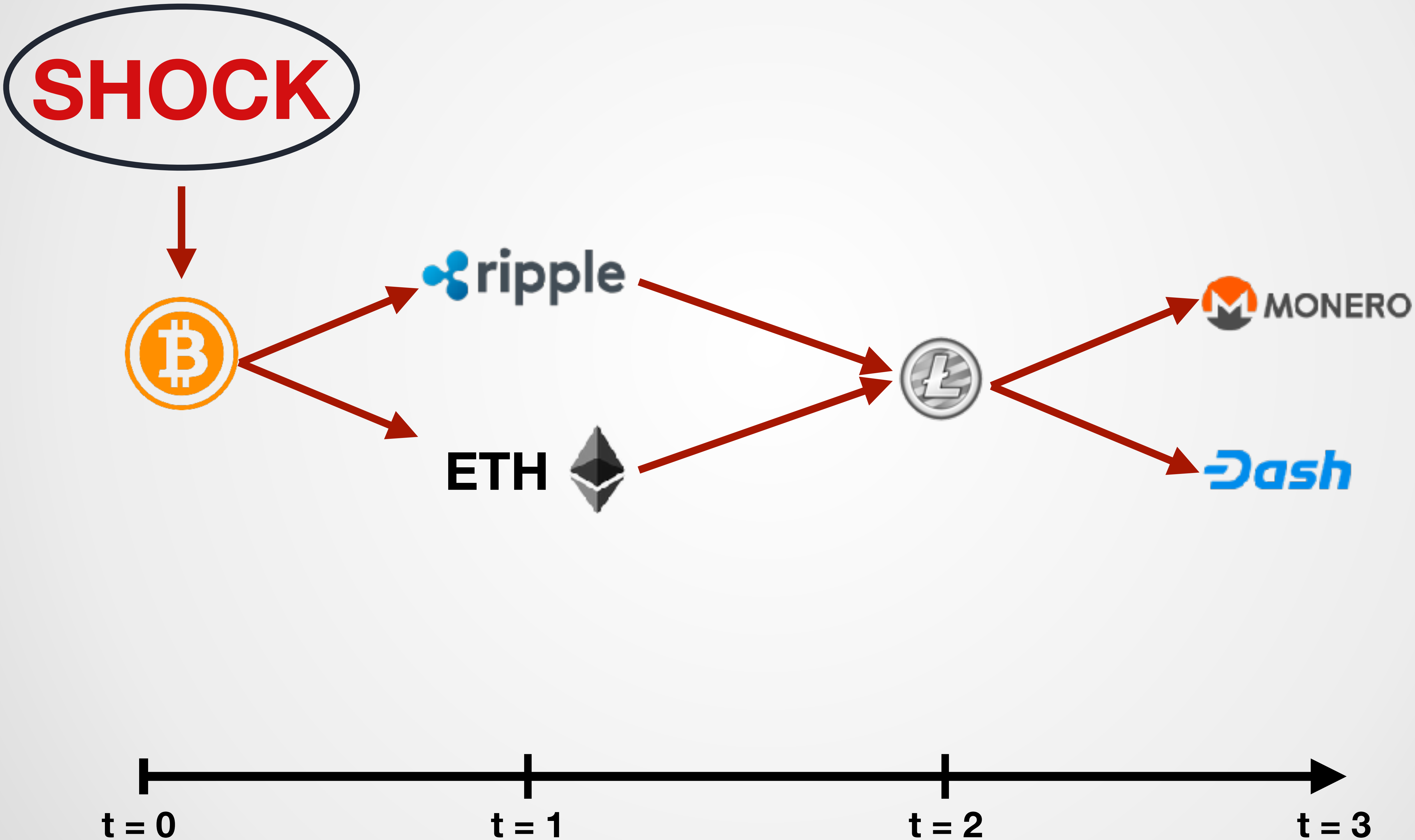
# Macro level: strong correlations


Dash, Litecoin, Monero

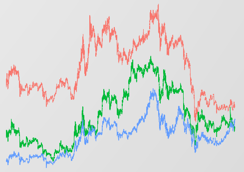
- Volatile despite market capitalization
- Quiet zones vs volatile zones
- Interconnectedness in joint movements?



# Example: contagion effects on a micro scale





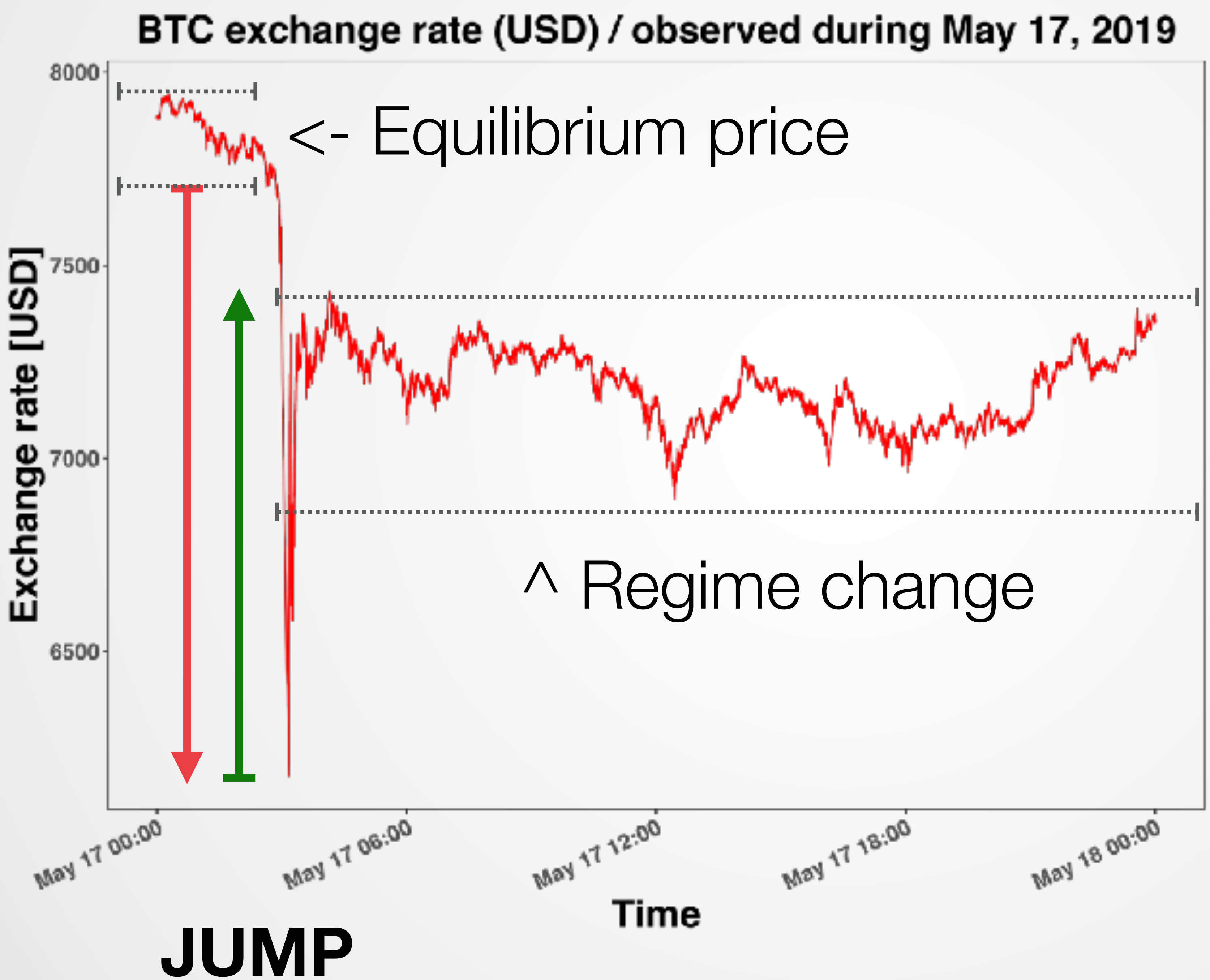
# Questions raised

- What is the structure of the underlying contagion dynamics in cryptocurrency markets?
- Can we identify patterns across currencies and exchanges?

Idea: predict contagion effect reactions

# High frequency financial data is challenging

A non-parametric model for estimation of true, unobserved price



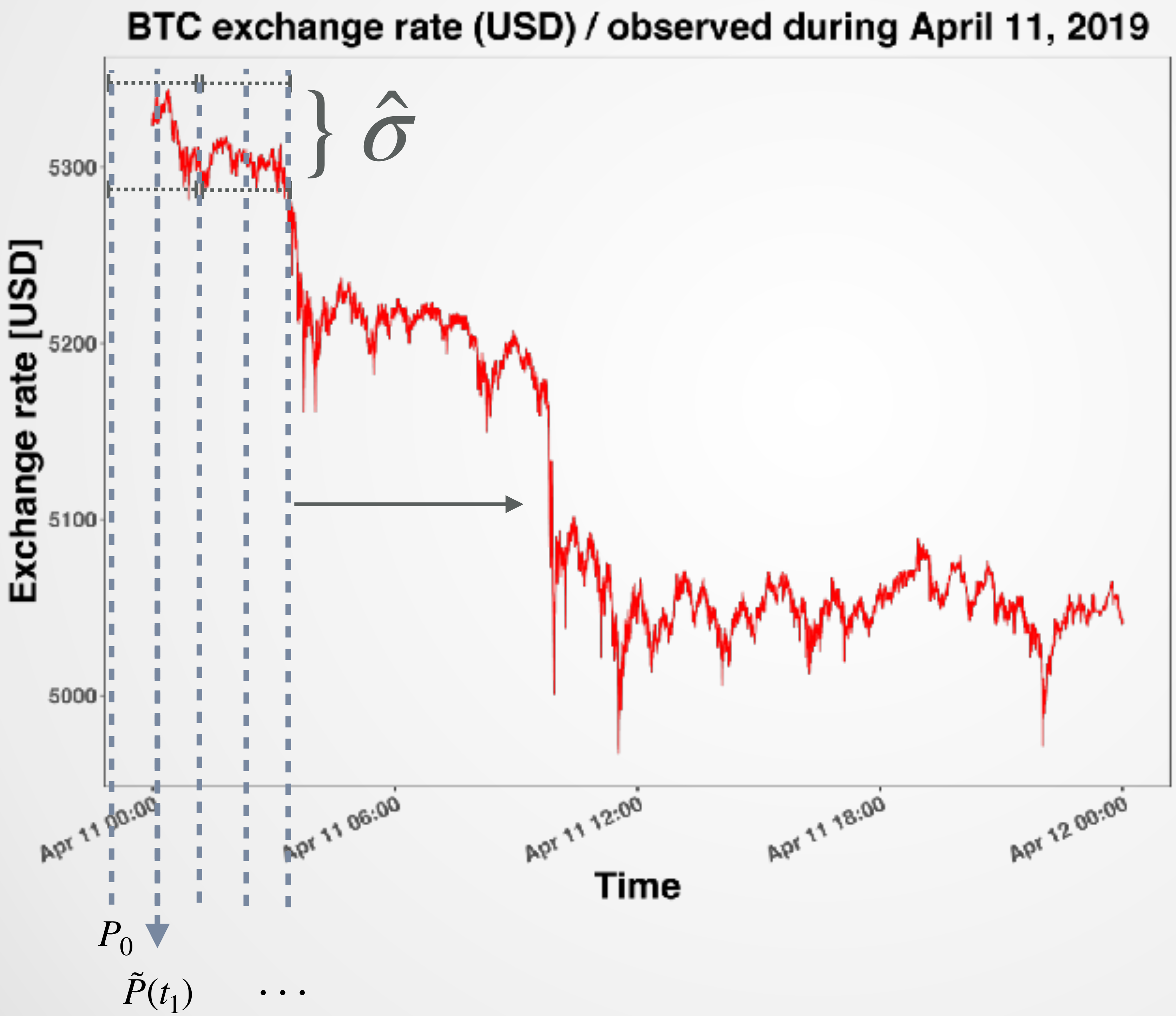

$$dX_t = \sigma dW_t + Y_t dJ_t$$

- $X_t \in \mathbb{R}$ - log price
- $\sigma \in \mathbb{R}^+$ - Volatility
- $W_t \in \mathbb{R}$ - Brownian Motion
- $J_t \in \{0,1\}$ - Jump arrival indicator
- $Y_t \in \mathbb{R}$ - Jump size

**Objective**: data-driven **finding** of $J_t$

# Identifying jumps - Lee / Mykland (2012)

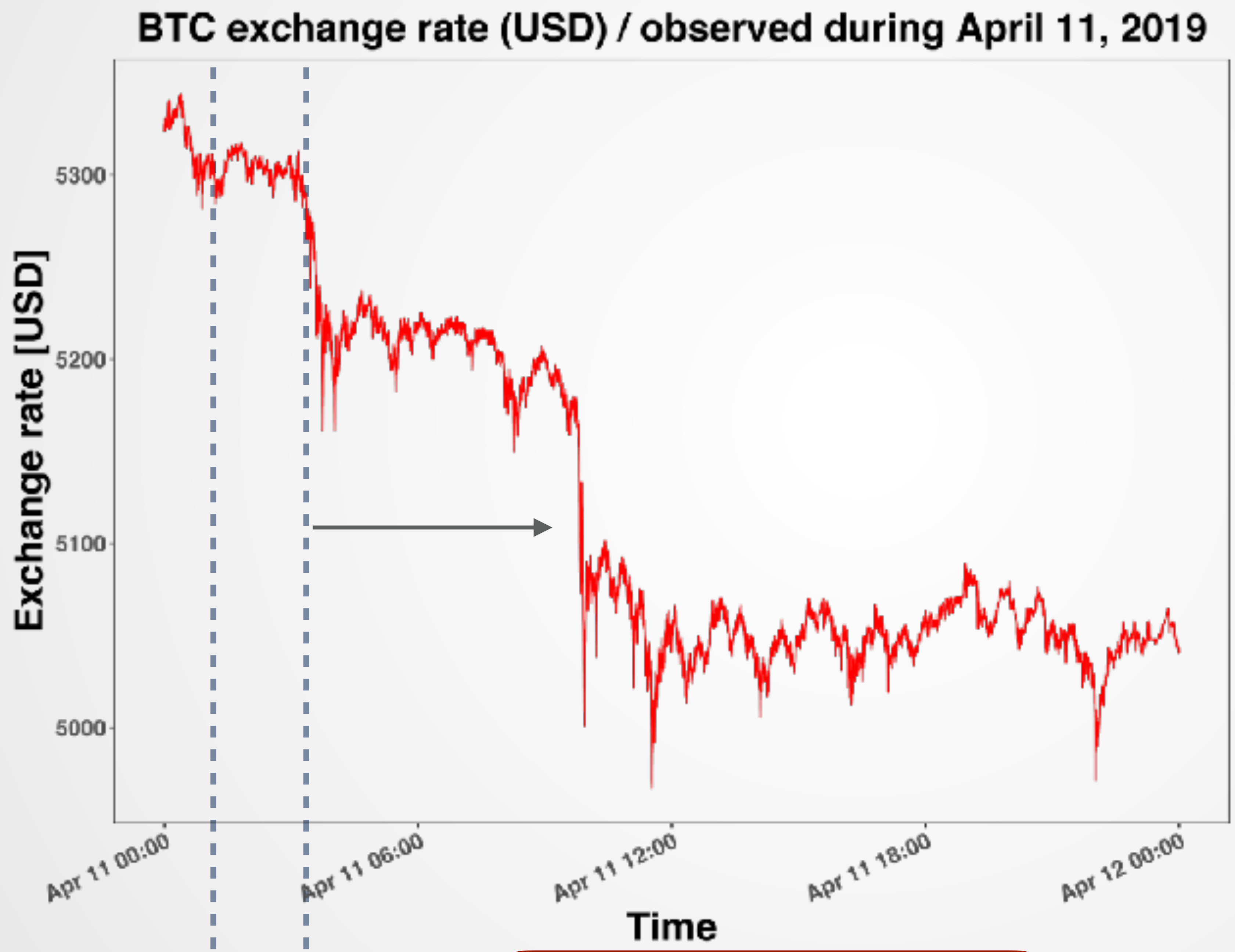
Sample price over $\mathcal{G}_n^k = \left\{0 = t_{n,0} < t_{n,k} < t_{n,2k} < \cdots\right\}$ over $[0,T]$




- $P_t = X_t + \epsilon_t$
- $\epsilon_t \in \mathbb{R}$
  - noise (serially correlated)
- $\hat{\sigma} \in \mathbb{R}^+$
  - volatility estimate
- $\tilde{P}(t_{ik})$ = subsampled price in $\mathcal{G}_n^k$
- $k \in \mathbb{N}$
  - ACF lag order

# Identifying jumps - Lee / Mykland (2012)

Introduce $\mathcal{G}_n^{kM} = \{t_n < t_{n,kM} < t_{n,2kM} < \cdots\} = \{t_0 < t_{kM} < t_{2kM} < \cdots\}$



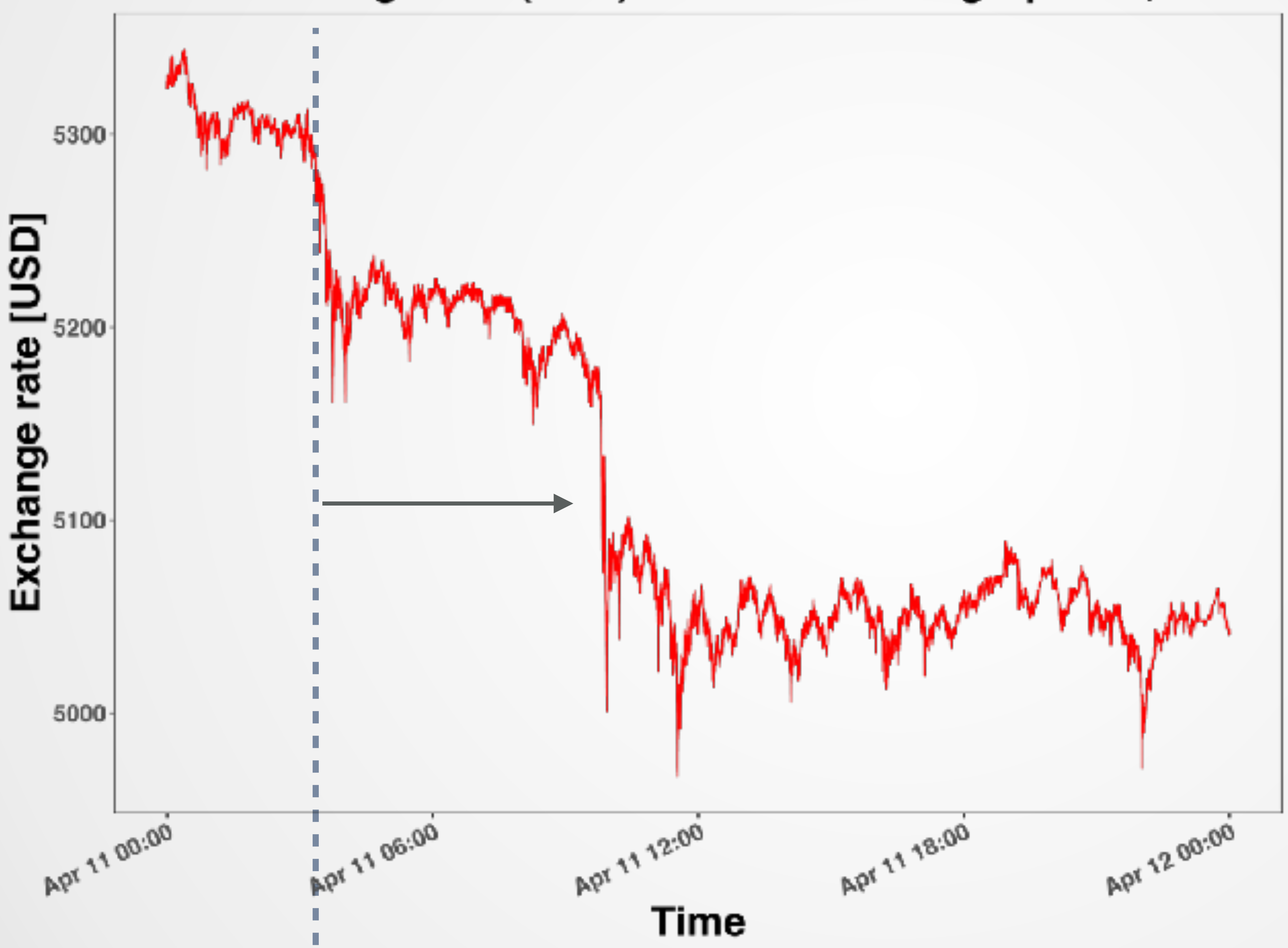

- Sample $\hat{P}(t_j)$ at every $M \in \mathbb{N}$ observations from $\mathcal{G}_n^k$
- $t_j \in \mathcal{G}_n^{kM}$ for all $j$
- $\hat{P}(t_j) \equiv \frac{1}{M} \sum_{i=\lfloor j/k \rfloor}^{\lfloor j/k \rfloor + M - 1} \tilde{P}(t_{ik})$
- $\mathcal{L}(t_j) \equiv \hat{P}(t_{j+kM}) - \hat{P}(t_j)$

$\hat{P}(t_{j=1})$ $\hat{P}(t_{1+kM})$ $\succ$ $\boxed{\hat{P}(t_{1+kM}) - \hat{P}(t_{j=1}) = \mathcal{L}(t_1)}$

# Identifying jumps - Lee / Mykland (2012)

For every $\mathscr{L}_{t_j}$ calculate test statistic $\hat{\xi}_{t_j} \equiv \dfrac{|\chi\left(t_j\right)| - A_n}{B_n}$ , $\xi$ ~ standard Gumbel distr.

**BTC exchange rate (USD) / observed during April 11, 2019**




$\hat{\xi}_1 \quad \cdots$

From Lee/Mykland (2012):

- $\dfrac{max_{t_j \in \mathscr{G}_n^{kM}} |\chi\left(t_j\right)| - A_n}{B_n} \xrightarrow{\mathscr{D}} \xi$
- $A_n \in \mathbb{R},\ B_n \in \mathbb{R},\ M \in \mathbb{N}$
- $\chi(t_j) \equiv \dfrac{\sqrt{M}}{\sqrt{V_n}} \mathscr{L}(t_j)$
- $V_n \equiv Var\left[\sqrt{M}\mathscr{L}(t_j)\right]$

# What are contagion dynamics? Can we find patterns?

To answer these questions, a new dataset has been collected:

- Discontinuous frequency (tick data)
- Aggregated to 1, 5, 10, 15 seconds for testing
    - Depending on no. of observations
    - Impute missing data
    - If less than 15 seconds: data is not „high frequency“ (definition of high frequency?)

Goal: **Ranking** of most common **contagion patterns** for modeling

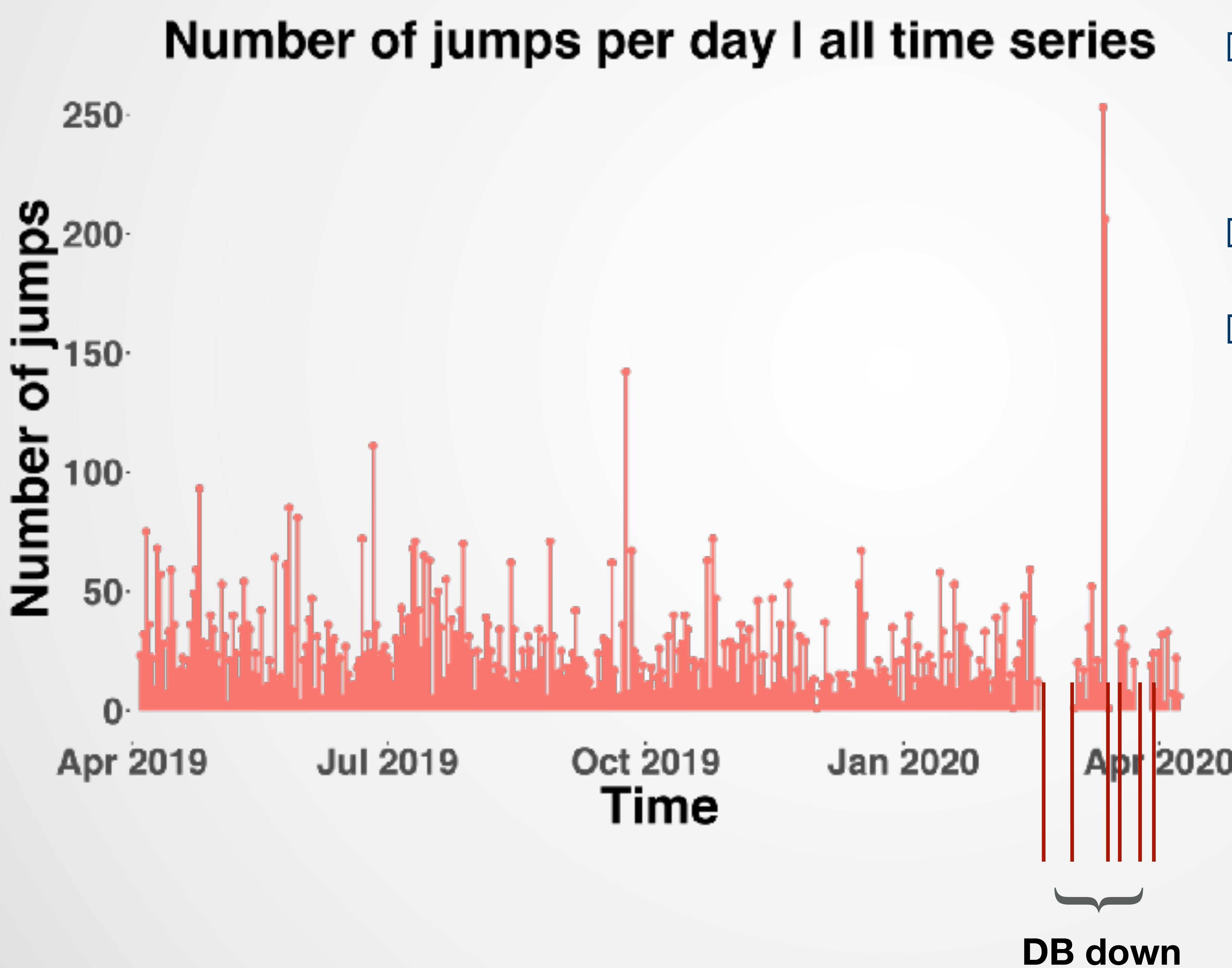
# Observations per exchange / currency

N = 500 mio.

- Binance
- OKEx
- Coinbase Pro
- Bitfinex
- HitBTC
- Bitstamp
- Poloniex

**April 2019** → **April 2020**

# Jumps are varying over time (example: $\alpha = 0.01$)




- Test every currency on every exchange / daily
- # time series: 15.617
- # jumps with $\alpha = 0.05$ (0.01, 0.001)
  - 42.097
  - 9.325
  - 2.392

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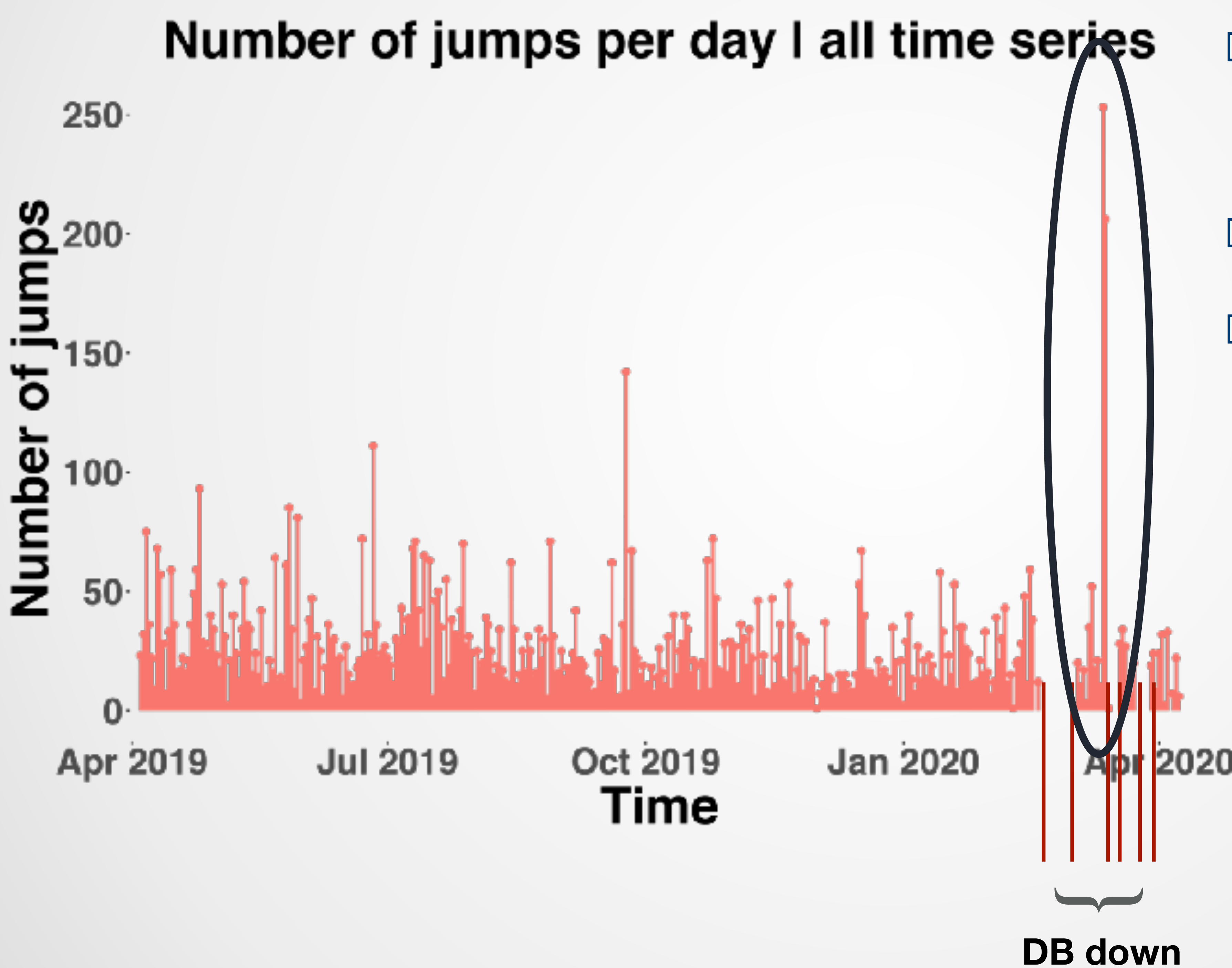


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- # time series: 15.617
- # jumps with $\alpha = 0.05$ (0.01, 0.001)
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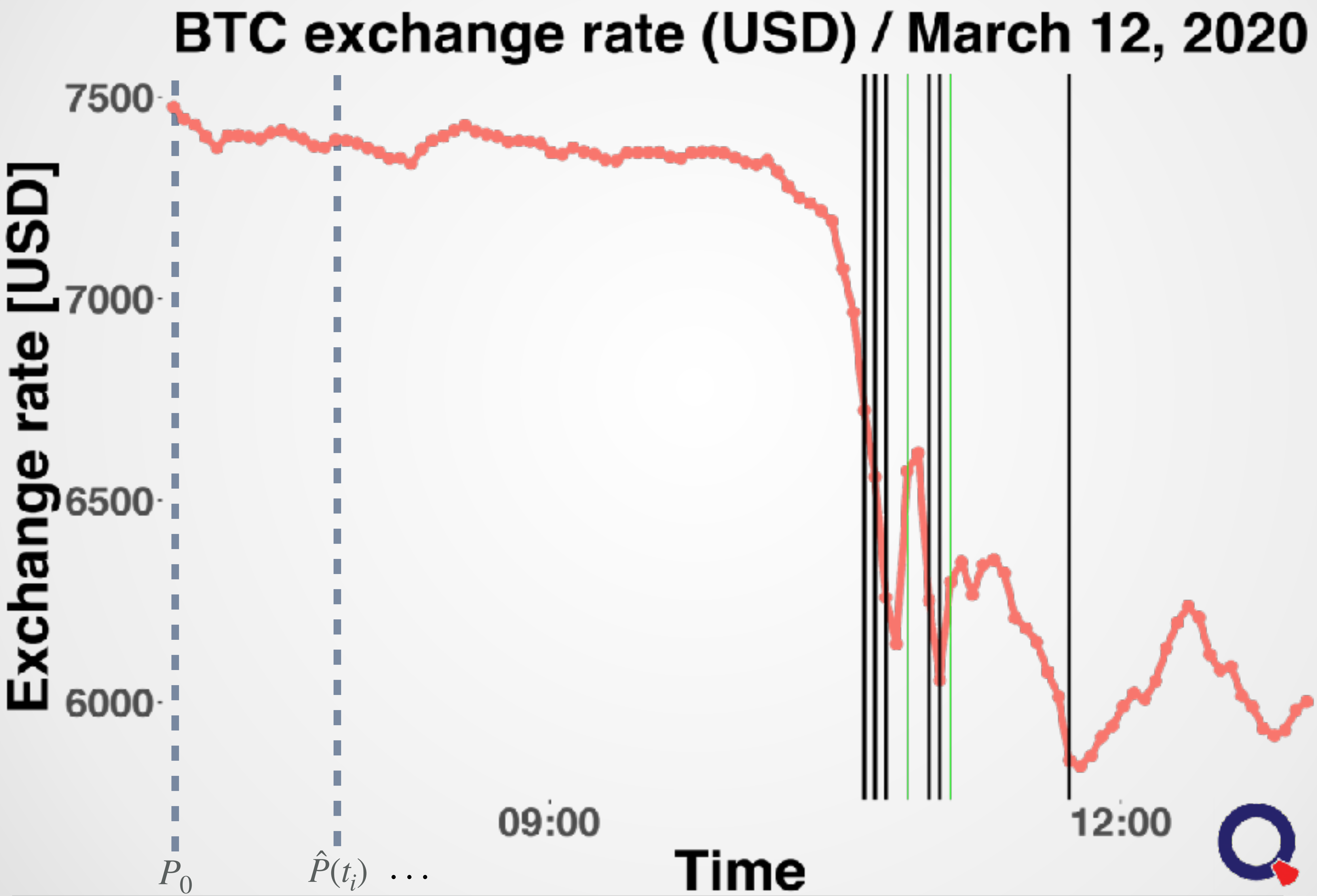
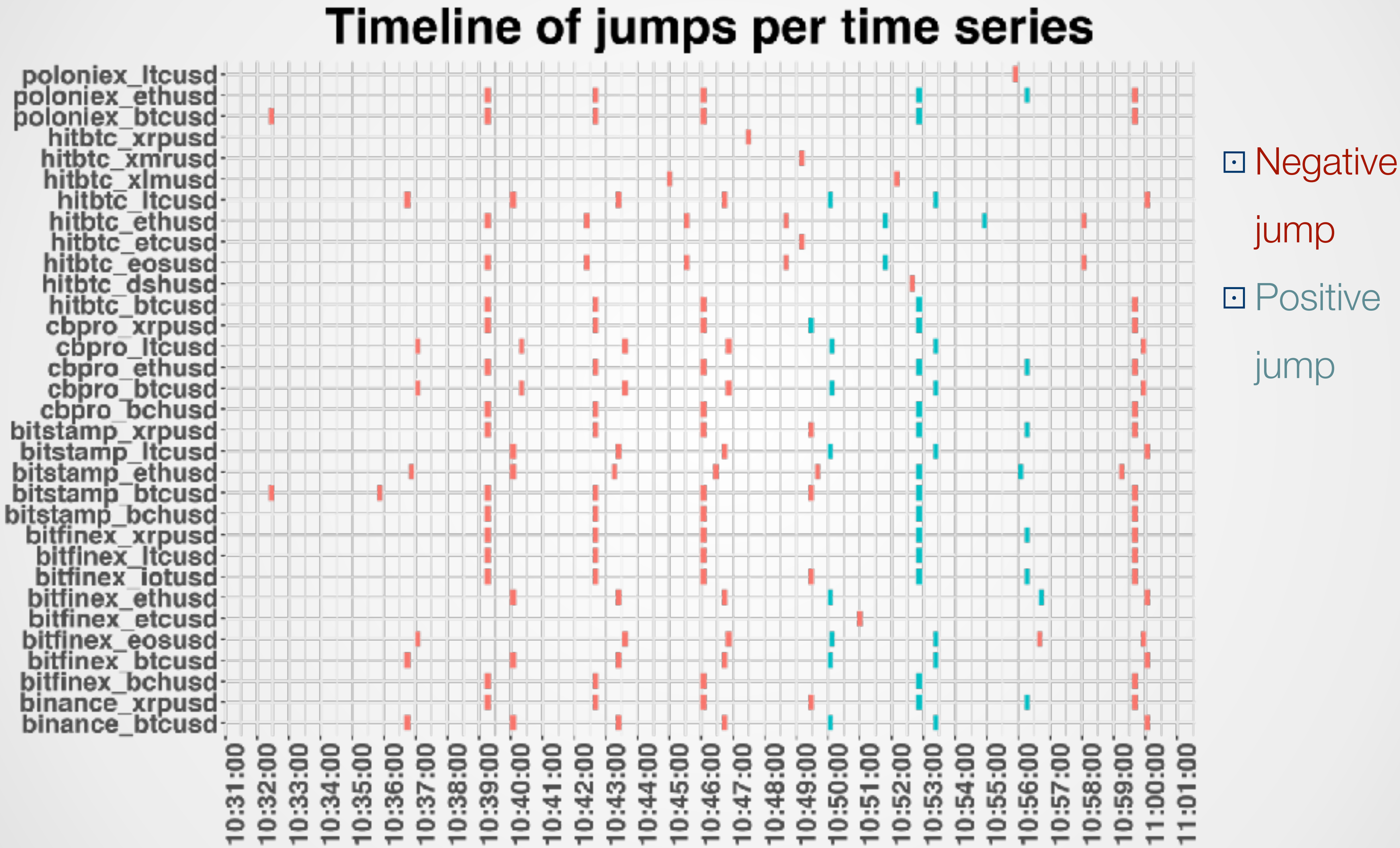
# Jumps can be clustered (example: Binance / $\alpha = 0.01$)



# Zooming in: spill over effects and waves ($\alpha = 0.01$)



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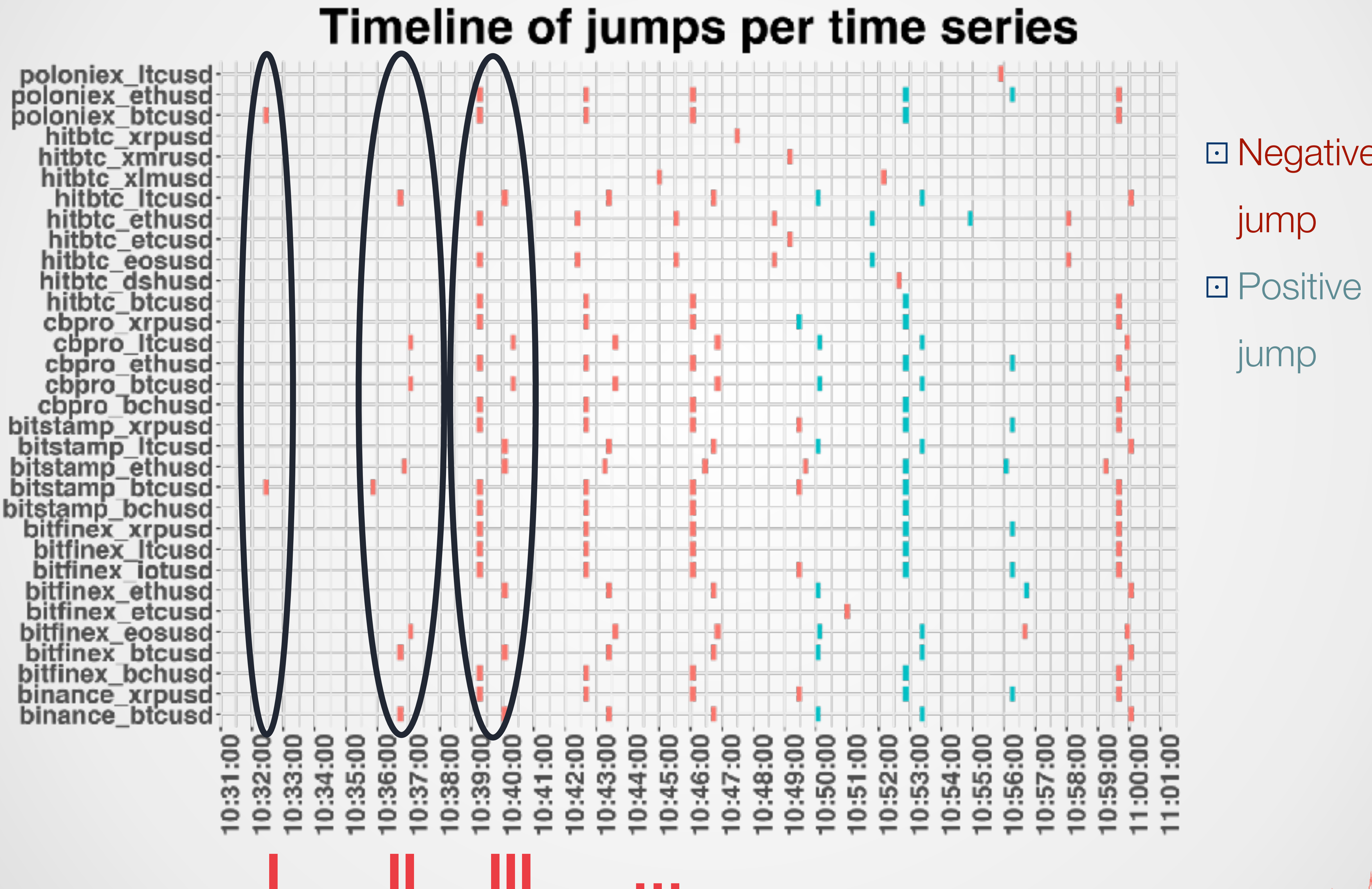

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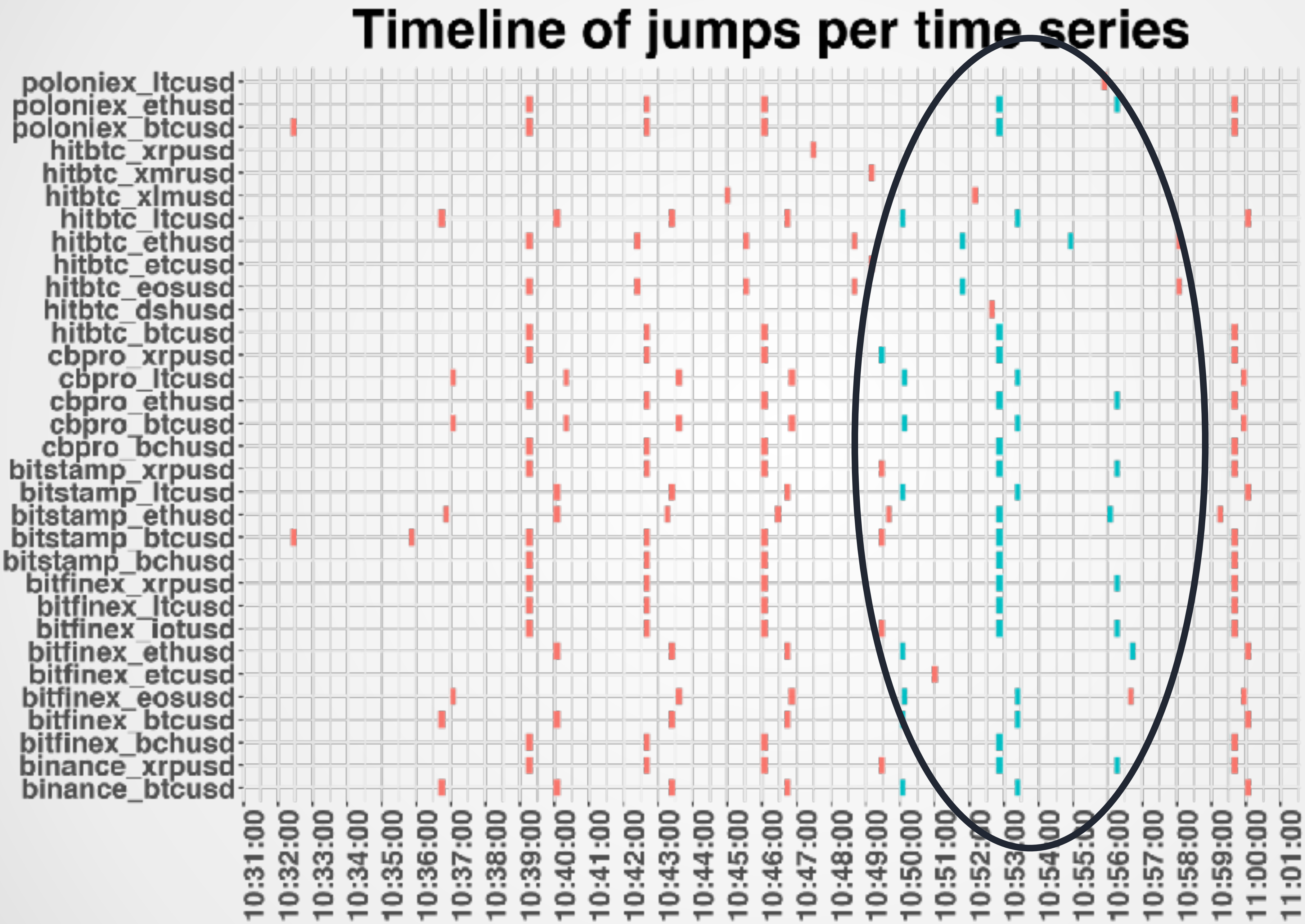
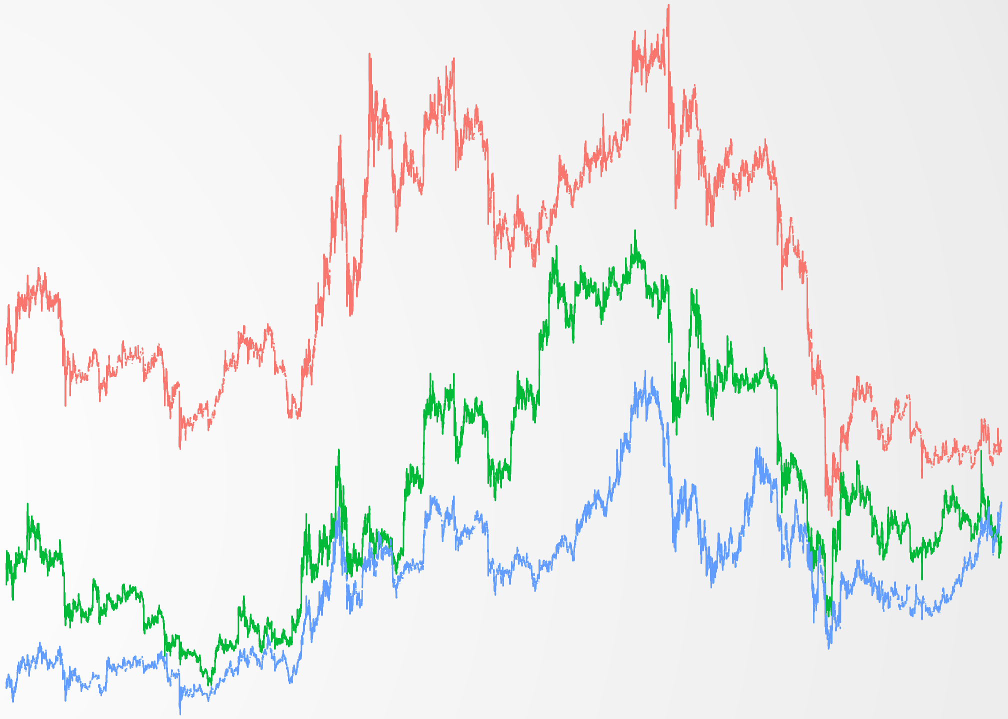



- Negative jump
- Positive jump

# Open problems: Modeling financial contagion after shocks in markets

- Exact jump time can only be approximated due to noise
    - Is it possible to identify first movers?
- How to model transitions between positive / negative jump waves?
    - Graph models?
    - Event trees?

# Jumps in cryptocurrencies



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- J. Jacod, Y. Li, P. A. Mykland, M. Podolskij, and M. Vetter. Microstructure noise in the continuous case: The pre-averaging approach. Stochastic Processes and their Applications, 119(7):2249–2276, July 2009.
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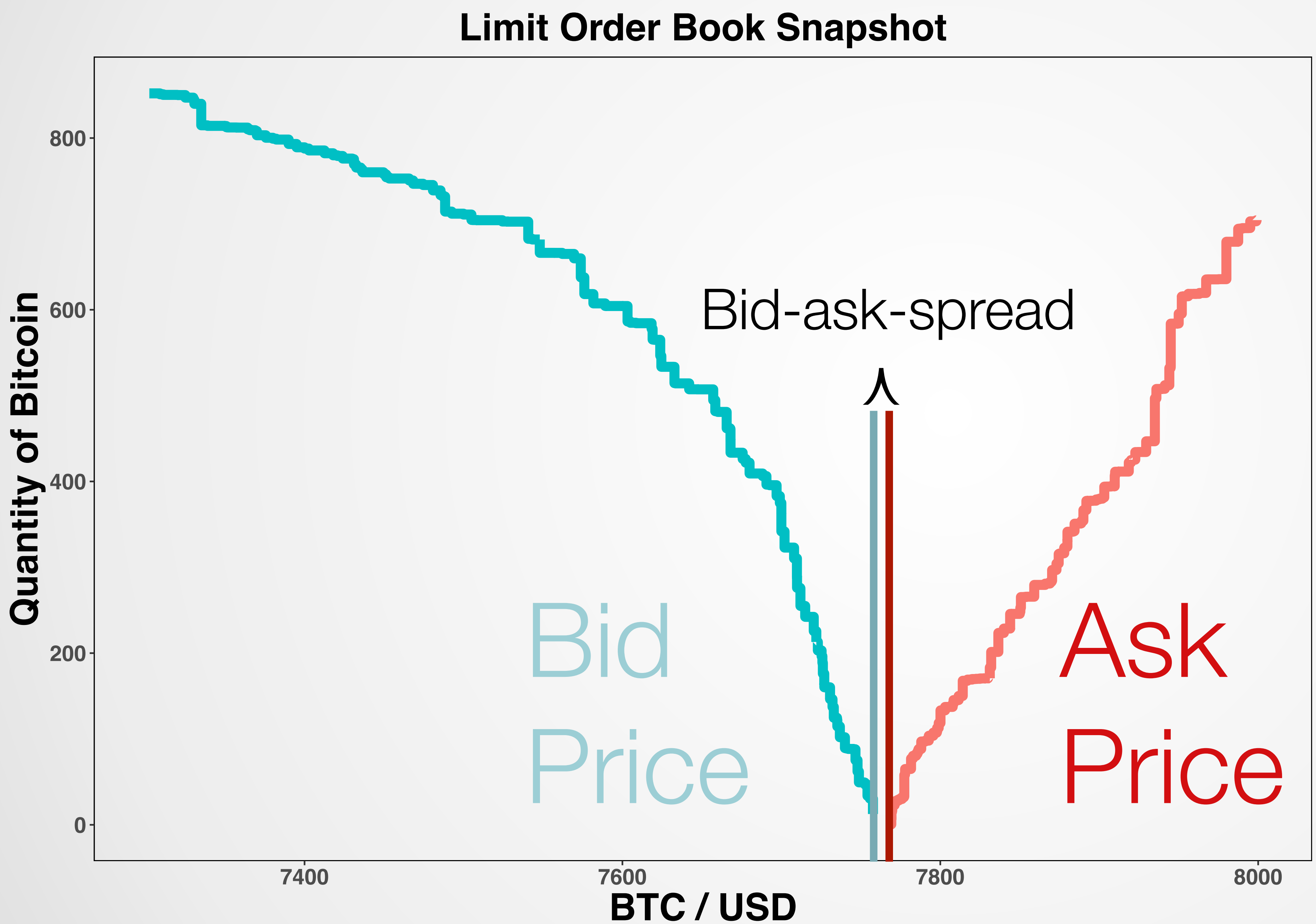
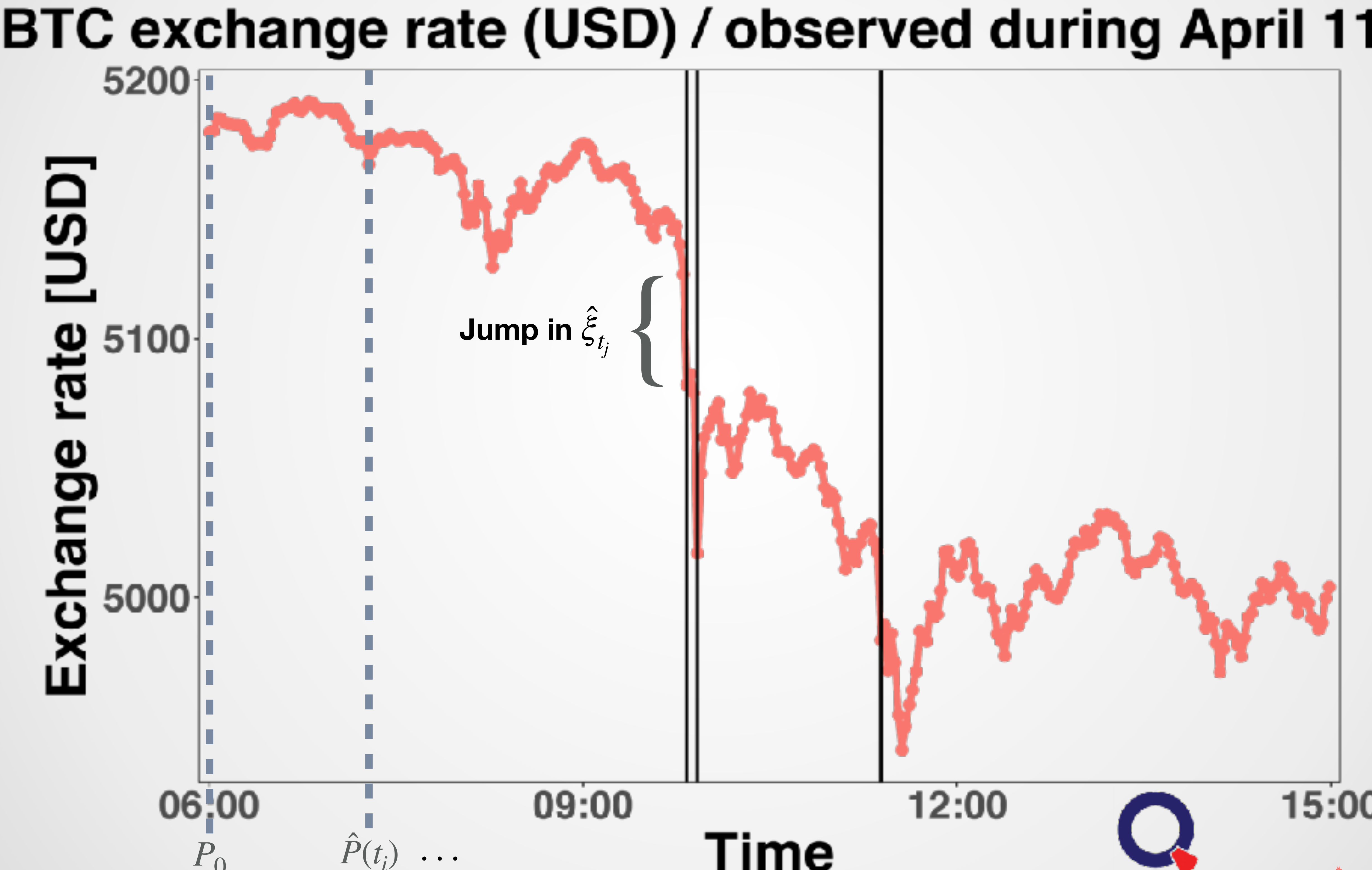
# High frequency financial data is challenging




- Agents post bid & ask prices
  - If prices change, agents may quickly revoke bids / asks
  - We observe current offers and past transactions
- Bid-ask-spread:
  - Impossible to observe true price
  - Large orders can trigger **jumps**

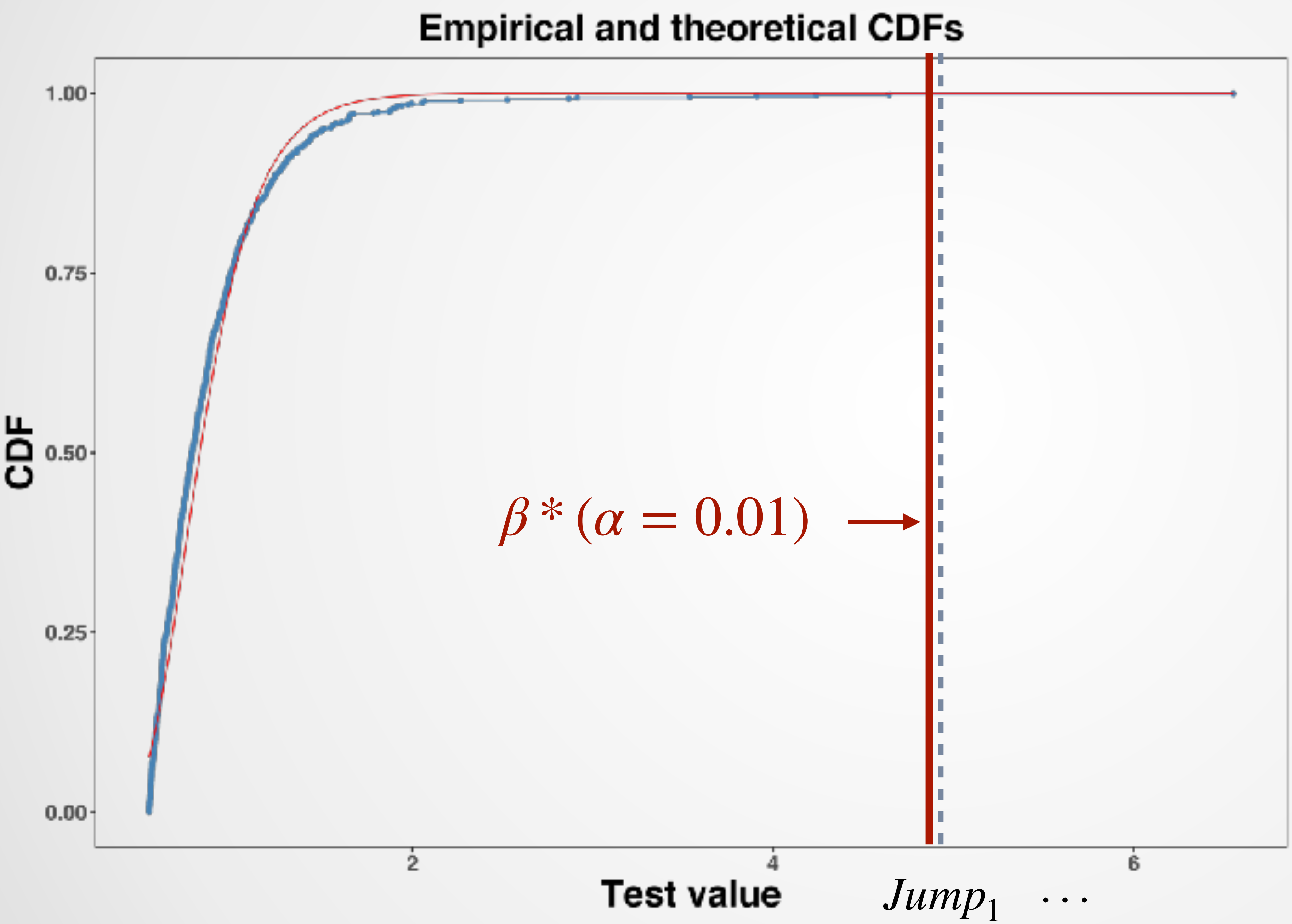
… limit order book & bid-ask-spread

# Example: jumps on April 11 - Lee / Mykland (2012)



# Example: jumps on April 11 - Lee / Mykland (2012)

Assumption: test statistic $\hat{\xi}$ follows Gumbel distribution, if e.g. $\hat{\xi}$ > 99th percentile $\succ$ jump




- Example: significance level of 99%
- Calculate $\hat{\xi}_{t_j}$ for every moment

Result: jumps **not present** can be **rejected** in multiple moments $\hat{\xi}$

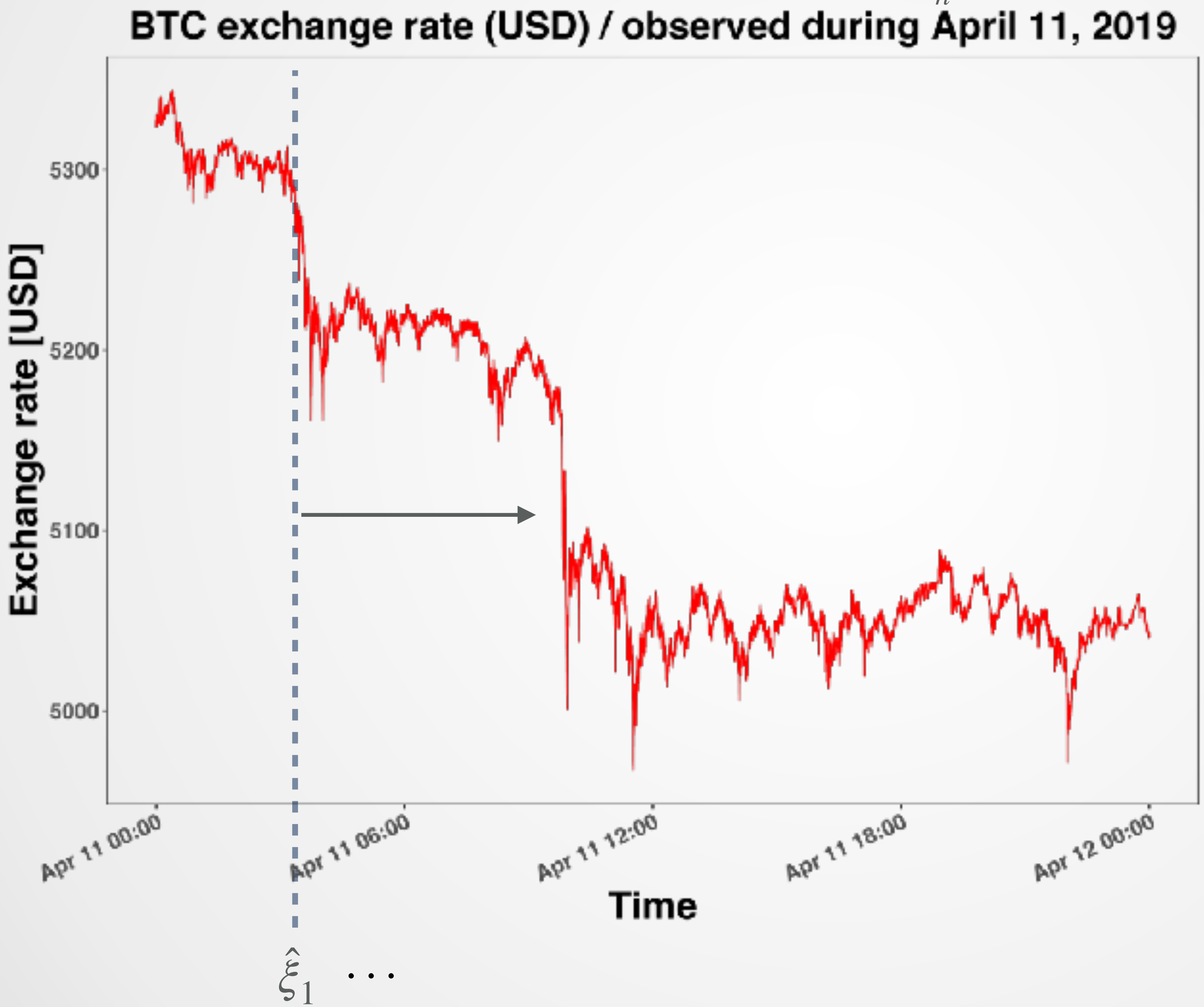
# Dataset: No. of observations per exchange / currency

| | |
|---|---|
| BTC | 241 Mio. |
| ETH | 70 Mio. |
| XRP | 47 Mio. |
| LTC | 37 Mio. |
| BCH | 21 Mio. |
| ETC | 18 Mio. |
| XLM | 8 Mio. |
| EOS | 8 Mio. |
| XMR | 4 Mio. |
| IOTA | 3 Mio. |
| ZRX | 2 Mio. |
| DSH | 2 Mio. |

| | |
|---|---|
| Binance | 242 Mio. |
| OKEx | 76 Mio. |
| Coinbase Pro | 52 Mio. |
| Bitfinex | 42 Mio. |
| HitBTC | 30 Mio. |
| Bitstamp | 14 Mio. |
| Poloniex | 5 Mio. |

# Identifying jumps - Lee / Mykland (2012)

For every $\mathscr{L}_{t_j}$ calculate test statistic $\hat{\xi}_{t_j} \equiv \frac{|\chi(t_j)| - A_n}{B_n}$, $\xi$ ~ standard Gumbel distr.




From Lee/Mykland (2012):

- $A_n = \left(2\log\left\lfloor\frac{n}{kM}\right\rfloor\right)^{1/2} - \frac{\log\pi + \log\left(\log\left\lfloor\frac{n}{kM}\right\rfloor\right)}{2\left(2\log\left\lfloor\frac{n}{kM}\right\rfloor\right)^{1/2}}$
- $B_n = \frac{1}{\left(2\log\left\lfloor\frac{n}{kM}\right\rfloor\right)^{1/2}}$
- $M \sim C\lfloor n/k\rfloor^{\frac{1}{2}}$